Wave Optics

- The disturbance of particles in a medium called wave.
- The locus of all the particles in the medium which are vibrating in the same phase is called wave front.

\[ \text{Wave front} \]

- Spherical wave front
- Cylindrical wave front
- Plane wave front

\[ \{ \text{Wave emits from point source.} \} \quad \{ \text{Wave emits from line source.} \} \quad \left\{ \begin{array}{l} \text{Small path of} \\ \text{spherical and} \\ \text{cylindrical wave front called} \\ \text{plane-wave front.} \end{array} \right\} \]

- Huygens Principle

\[ S \text{ be the source.} \]

- Every point on a given wave front may be regarded as a source of new disturbance.
- The new disturbance from each point spread out in all directions with a velocity of light are called secondary wave front.
- The source of tendency to the secondary wave front is forward & backward direction in time t.
In fig. $x', y'$ is called forward secondary wave front and $x'', y''$ is called backward secondary wave front.

Law of Reflection in wave optics.

$AB$ is the reflecting surface.

$AL$ and $BM$ are the incident ray $L$ reflected ray.

Draw perpendiculars from $L$ to $O$ to and $M$ to $O$.

Total Time $= \frac{OL}{v} + \frac{OM}{v}$

In $\Delta ALO$,

$$\sin i = \frac{OL}{OA}$$

In $\Delta OMB$,

$$\sin r = \frac{OM}{OB}$$

Total Time $= \frac{OA \sin i}{v} + \frac{OB \sin r}{v}$

$$= \frac{OA \sin i + (AB - OA) \sin r}{v}$$

$$= \frac{OA}{v} \left[ \frac{\sin i - \sin r}{\sin r} \right] + \frac{AB \sin r}{v}$$

Condition:

Different incident ray have different value of $OA$ but time is same for all. It is only possible when the product of $OA$ is zero ($0$)
\[
\frac{\sin i - \sin r}{V} = \frac{\sin i - \sin r}{V} = 0
\]

\[
\sin i = \sin r
\]

\[
i = r
\]

- **Law of Refraction**

\[
AB = \text{Refracting surface.}
\]

\[
\text{Total time} = \frac{OL}{V_1} + \frac{OM}{V_2}
\]

\[
\sin \angle OAL = \frac{OL}{OA}
\]

\[
OL = OA \sin i
\]

\[
\sin \angle OBM = \frac{OM}{OB}
\]

\[
OM = OB \sin \theta
\]

\[
\text{Total time} = \frac{OA \sin i}{V_1} + \frac{OB \sin \theta}{V_2}
\]

\[
= \frac{OA \sin i}{V_1} + \frac{(AB - OA) \sin \theta}{V_2}
\]

\[
= OA \left[ \frac{\sin i}{V_1} - \frac{\sin \theta}{V_2} \right] + \frac{AB \sin \theta}{V_2}
\]
Condition:

- Different incident rays have different values of OA but time is same for all. It is only possible when the product of OA is zero.

\[
\frac{\sin i_1}{\sin i_2} = \frac{V_1}{V_2}
\]

\[
\frac{\sin i_1}{\sin i_2} = \frac{V_1}{V_2} = \frac{C/n_1}{C/n_2} = \frac{n_2}{n_1}
\]

\[
\frac{\sin i_1}{\sin i_2} = \frac{n_2}{n_1}
\]

\[
n_1 \sin i_1 = n_2 \sin i_2
\]

Interference:

When two waves superimpose each other, at one time we get maximum intensity of light and at another time we get minimum intensity of light. When crust trough falls on smooth surface and at another time we get minimum intensity of light when crust falls on rough surface, then bright and dark phases form. This is called interference.

Condition for constructive and destructive interference

\[Y_1 = a_1 \sin \omega t\]

\[Y_2 = a_2 \sin (\omega t + \phi)\]

According to superposition, imposition. (\[A_1A_2 + S_1P\])

\[Y = Y_1 + Y_2\]
\[ y = a_1 \sin \omega t + a_2 \sin(\omega t + \phi) \]
\[ = a_1 \sin \omega t + a_2 \left[ \sin \omega t \cos \phi + \cos \omega t \sin \phi \right] \]
\[ = \sin \omega t \left( a_1 + a_2 \cos \phi \right) + a_2 \cos \omega t \sin \phi \]

Put \[ a_1 + a_2 \cos \phi = A \cos \theta \] \[ a_2 \sin \phi = A \sin \theta \]

\[ y = A \sin \omega t \cos \theta + A \cos \omega t \sin \theta \]
\[ y = A \sin (\omega t + \theta) \]

Eqn 3' is called Resultant disturbances of wave.

\[ A^2 (\sin^2 \theta + \cos^2 \theta) = a_1^2 + a_2^2 \cos^2 \phi + 2a_1a_2 \cos \phi \]
\[ A^2 = a_1^2 + a_2^2 (\cos^2 \phi + \sin^2 \phi) + 2a_1a_2 \cos \phi \]
\[ A^2 = a_1^2 + a_2^2 + 2a_1a_2 \cos \phi \]

\[ I < A^2 \]
\[ I = I_1^2 + I_2^2 + 2I_1I_2 \cos \phi \]

For constructive interference \( \cos \phi = +1 \)

\[ \cos \phi = \cos 0^\circ, \cos 2\pi, \cos 4\pi \ldots \]

\[ \phi = 2n\pi \]

where \( n = 0, 1, 2, 3, \ldots \)

Hence, whole number multiply by phase difference of \( 2\pi \).
\[ \lambda \text{ path diff.} = 2\pi \text{ phase diff.} \]

1 phase difference = \[\frac{\lambda}{2\pi}\] path diff.

\[ \text{path diff} = \frac{\lambda}{2\pi} - 2\pi n \xi = m\lambda \]

For destructive interference, \( \cos \phi = -1 \)

\[ \phi = (2n-1)\pi \]

where \( n = 1, 2, 3 \)

Hence, odd no. multiplied with phase difference of \( \pi \)

\[ \text{path diff} = \frac{\lambda}{2\pi} (2n-1)\pi = \frac{\lambda}{2} (2n-1) \]

Hence odd no is multiplied with path difference of \( \frac{\lambda}{2} \)

This is known as destructive interference.

\[ \cos \theta = +1 \]

\[ \cos \theta = -1 \]

\[ l_{\text{max}} = a_1^2 + a_2^2 + 2a_1a_2 \]

\[ = (a_1 + a_2)^2 \]
Young's double slit Expt →

Cohesive sources →
Two sources are said to be cohesive if they have equal in amplitude, equal in phase diff. and equal in path diff. is said to be cohesive.

1 Real source and a virtual source. They are said to be cohesive.

Two virtual sources are said to be cohesive.
Expression for Fringe width 

Distance between two bright fringes:

\[ d = \sqrt{D^2 + \left(2y\lambda\right)^2} \]

\[ d \approx \frac{2y\lambda}{D} \]

For constructive interference:

\[ \frac{y}{D} = n\lambda \]

Hence:

\[ y = \frac{n\lambda D}{D} \]
For 1st bright range fringe:

\[ n = 1 \quad \gamma_1 = \frac{\lambda D}{d} \]

\[ n = 2 \quad \gamma_2 = \frac{2\lambda D}{d} \]

Hence fringe width is equal to distance between two consecutive bright fringes.

\[ \beta = \gamma_3 - \gamma_2 = \frac{3\lambda D}{d} - \frac{2\lambda D}{d} = \frac{\lambda D}{d} \]

For destructive interference:

\[ \frac{y_d}{D} = \frac{(2n-1)\lambda}{2} \]

\[ y = \frac{(2n-1)\lambda D}{2d} \]

For 1st dark fringe:

\[ n = 1 \quad \gamma_1 = \frac{\lambda D}{2d} \]

\[ n = 2 \quad \gamma_2' = \frac{3\lambda D}{2d} \]

\[ n = 3 \quad \gamma_3' = \frac{5\lambda D}{2d} \]

\[ \beta = \gamma_2' - \gamma_2' = \frac{5\lambda D}{2d} - \frac{3\lambda D}{2d} = \frac{\lambda D}{d} \]
Interference and conservation of energy.

Consider two sources of light emitting light waves \( a_1 \) and \( a_2 \) respectively. If there is no interference between light waves then intensity of light equal \( i = i_1 + i_2 = a_1^2 + a_2^2 \)

In case the light from two source interference the maxima and minima will be formed.

For maxima:
\[
I_{\text{max}} = (a_1 + a_2)^2
\]
\[
I_{\text{min}} = (a_1 - a_2)^2
\]
\[
I_{\text{ave}} = \frac{I_{\text{max}} + I_{\text{min}}}{2}
\]
\[
= (a_1 + a_2)^2 + (a_1 - a_2)^2
\]
\[
= a_1^2 + a_2^2 + 2a_1a_2 + a_1^2 + a_2^2 - 2a_1a_2
\]
\[
= 2(a_1^2 + a_2^2)
\]
\[
= a_1^2 + a_2^2
\]

**Diffraction of light:**

Diffraction: The phenomenon of bending of light around the sharp corner and spreading into the region of geometrical shadow is called diffraction.

**Diffraction of light is of two types:**

1) Fresnel diffraction.
2) Fraunhofer diffraction.
1) Fresnel diffraction -
   It is a type of diffraction that takes place at a slit when the source of light is at a finite distance from it.

2) Fraunhofer diffraction -
   It is a type of diffraction that takes place at a slit when a plane wave front is from incident on it and the wave front emerging from the slit is also plane.

Diffraction at a single slit -

L₁ and L₂ are converging lens.

Central maxima formed at O.

Width of central maxima is double of secondary maxima.

BN is path difference between two slits.

\[ \sin \theta = \frac{BN}{AB} = \frac{1}{A} \]
let $S$ be the source pass across converging lens $L_1$, and after cross it rays of light parallel which with pass across slit AB and pass across strike at converging lens $L_2$, after crossing across $L_2$, Central maxima formed at O in the screen surface.

The width of secondary central maxima is twice of secondary maxima. Intensity of secondary maxima goes on decreasing. At point O at central maxima ray of light is in the same phase when a plate is tilted with an angle $\theta$, then fringes formed at which has difference

Draw $1\sigma$ from A to N. Then $AN$ be the path diff. hence in $\triangle ABN$

$$\sin \theta = \frac{AN}{AB} = \frac{\lambda}{A}$$

Suppose that point $P$ on the screen is at such a distance from the centre of screen then $AN = \lambda$ and $\theta = \theta_1$

Hence $\sin \theta_1 = \frac{\lambda}{A}$

Thus is the position of 1st Secondary minima.

$AN = 2\lambda$

$\theta = \theta_2$

$$\sin \theta_2 = \frac{2 \lambda}{A}$$

For $n^{th}$ fringes,

$$\sin \theta_n = \frac{n \lambda}{A}$$
If \( y_n \) is the distance of \( n \)th minima from the center of the screen and \( d \) is the difference between slit and the screen, then:

\[
\tan \theta_n = \frac{y_n}{D}
\]

For smaller angle

\[
\sin \theta_n = \tan \theta_n
\]

\[
\frac{n^2}{\lambda} = \frac{y_n}{D}
\]

\[
y_n = nD\lambda
\]

For 1st minima, \( n = 1 \)

\[
y_1 = \frac{D\lambda}{\alpha}
\]

\[
n = 2, \quad y_2 = 2\frac{D\lambda}{\alpha}
\]

Breadth width = \( y_3 - y_2 \)

\[
\alpha = 3\frac{D\lambda}{\lambda} - 2\frac{D\lambda}{\lambda}
\]

\[
\alpha = \frac{D\lambda}{\lambda}
\]

For maxima, if point \( P \) on the screen is at such a distance from point \( O \) then

\[
\sin \theta' = \frac{3\lambda}{2a}
\]

This is the case of first secondary maxima.

Similarly,
\[ \sin \theta_n = \frac{\lambda}{2a} \]

For odd fringes:
\[ \sin \theta_n = (2n+1)\frac{\lambda}{2a} \]

For even fringes:
\[ \sin \theta_n = \frac{\lambda}{2a} \]

For smaller angles, \( \theta_n \approx \tan \theta_n \)
\[ \frac{(2n+1)\lambda}{2a} = \frac{\lambda}{2a} \]

For 1st maxima, \( n = 0 \)
\[ y_1 = \frac{3\lambda D}{2a} \]

For 2nd maxima, \( n = 1 \)
\[ y_2 = \frac{\lambda D}{2a} \]

For 3rd maxima, \( n = 2 \)
\[ y_3 = \frac{7\lambda D}{2a} \]

Fringe width: \( y_3 - y_1 \)
\[ y_3' = y_3 - y_1 = \frac{7\lambda D}{2a} - \frac{3\lambda D}{2a} = \frac{4\lambda D}{2a} = \frac{2\lambda D}{a} \]
Point of difference in both Interference & Diffraction.

1. Interference is the result of interaction of light coming from two wave fronts originating from two coherent sources.

Diffraction is a pattern result of interaction of light coming from different paths of same wave front.

2. In interference, the fringes may be may not be of same width.

In diffraction, the fringes are always of varying width.

3. In interference the fringes of minimum intensity are perfectly dark.

In diffraction the fringes of minimum intensity are not perfectly dark.

4. In interference, all the bright fringes are of same intensity.

In diffraction the bright fringes are of varying intensity.

5. In the interference, there is good contrast between bright & dark fringes.

In diffraction pattern, the contrast between bright and dark fringes are comparatively poor.
Resolving power of microscope.
It is defined as the reciprocal of the least separation between two close objects so that they appear just separately when seen from microscope.

The least separation between the two objects.

\[ d = \frac{\lambda}{2D \sin \theta} \]

where \( \lambda \) is wavelength, \( D \) is refractive index, and \( \theta \) is the semi-vertical angle made by light of objective and cone of microscope.

The least distance is called limit of resolution of microscope.
Hence, resolving power of microscope is equal to reciprocal of limit of \( \frac{1}{d} \) resolution.
Hence, resolving power of microscope = \( \frac{1}{d} \)

\[ \frac{1}{d} = \frac{2D \sin \theta}{\lambda} \]

Resolving power of a telescope.
It is defined as the reciprocal of the smallest angular separation between two distant objects so that they appear just separated when seen through the telescope.

The smallest angular separation between the two objects so that they appear just separated.

\[ d \theta = \frac{1.22 \lambda}{D} \]

where \( D \) is the diameter of the objective, \( \lambda \) is the wave length of light and \( d \theta \) is the angular separation.
Since the resolving power of a telescope is equal to reciprocal of limit of angular resolution, hence resolving power of microscope

\[
\frac{1}{d\theta} = \frac{D}{1.22\lambda}
\]

* Polarisation of light.

Transverse nature of light.

The phenomenon due to which the vibration of light is restricted in a vertical plane is called polarisation of light.

Plane of vibration: The plane ABCD which contains the vibration of plane polarised light is called plane of vibration.

Plane of polarisation: The plane PARS i.e. to the plane of vibration is called the plane of vibration.
Plane Polarised Light.

It may be defined as the light in which the vibration of the light are restricted to a particular plane. According to electromagnetic theory, the electric vector acts as a light vector. Therefore, in a plane polarised light, the electric vector vibrates along a fixed straight line in a plane perpendicular to the direction of propagation.

To detect the plane polarised light:

It cannot be detected through the naked eyes.

To detect the plane polarised light, we take two crystals, one is polariser and the other is analyser. When passing the light through the one and after vibrating the light passes through the another, which rotates and after that, the intensity of light starts decreasing and the light falls to the polarised light.

Polarisation by Reflection:

When a ray of light passes across a glass slab, then some rays are reflected and refracted and some rays are reflected. Then the reflected ray is called plane polarised light and angle both the refracted and reflected rays is 90°. And that particular angle of incidence is known as angle of polarization.
Brewster’s law \[ \theta = \tan \phi \]

In case of plane polarised light refractive index equal to tangent of angle of polarisation.

\[ \frac{\sin i}{\sin r} = \frac{\sin \phi}{\cos \phi} \]

\[ \sin r = \cos \phi = \sin (90 - \phi) \]

\[ \alpha = 90 - \phi \]

\[ \alpha + \phi = 90^\circ \]

It shows that when ray of light incident at polarising angle, the reflected ray is at right angle to the refracted ray.
Polariisation by Scattering.

When a beam of light passes through a medium it gets scattered from the particles constituting the medium provided the size of the particle is of the order of wavelength of light. The scattered light viewed in the direction perpendicular to the beam of light is called polarisation of light.

In fig. unpolarised light is along the x-axis, after scattering, light scatter along y-axis, in which vibration of light is along x-axis and z-axis.

When 
- x to y axis then parallel to z-axis.
- y to z axis parallel to y-axis.

Law of Malus:-

It states that when a completely plane polarised light beam is incident on analyser then intensity of emergent light varies as a square of cosine of the angle between the plane of transmission of the analyser and the polariser.
consider that plane of polariser and plane of analyser inclined at an angle $\theta$.

Plane Polarised light of Intensity $I_0$ and amplitude $A$ incident on the polariser. Then amplitude $a$ has two components. The component $a \cos \theta$ along the plane of transmission of analyser and component $a \sin \theta$ in a direction perpendicular to the plane of analyser.

In case, there is no case of light due to absorption then,

$$I \propto (a \cos \theta)^2$$

$$I = I_0 \cos^2 \theta$$

$I$ is proportional to $\cos^2 \theta$, is called law of Malus.

**Special case:**

1) when $\theta = 0$ or $180^\circ$,
then $\cos \theta = \pm 1$
so, $I = I_0$

There, when polariser and analyser are parallel, the intensity of light transmitted by the analyser is same as that falls on it from the polarised.

2) when $\theta = 90^\circ$
then $\cos \theta = 0$
so, $I = 0$

Therefore, when polariser and analyser are at right angles, the intensity of light transmitted from the analyser is zero i.e. minimum.
If you plot a graph between intensity of light and the angle between analyzer and polarizer, it is shown in the fig.

\[ I = \frac{1}{2} \cos^2 \theta \]

In case, the light incident on polarizer is unpolarized, then \[ I = \frac{1}{2} \] beq average value of \[ \cos^2 \theta \]

Polarizers:
Polarizers are large and thin sheet of crystalline polarizing material capable of producing plane polarized beams of large cross-section.

Uses of Polarizers:
- In sunglasses.
- In windshields of automobiles.
- In window pans.
- In three dimensional motion picture.
Sol — Solid in liquid.
   eg: paint, cell fluid.

Gel — Liquid in solid.
   eg: gelatine, butter.

Alcohol — dm is alcohol.

Hydrosol — dm is water.

Solid sol — Coloured glasses.

Adsorption

Leophbic sol — Liquid loving sol.
   For eg: starch, gum, gelatin, rubber.

Properties
i) Stable
ii) Reversibility in nature.
iii) Coagulation does not happen.

Leophotic sol — Liquid hating metals.
   eg: metallic sol.

Properties
i) Irreversible
ii) Unstable.

CMC — A minimum conc. above which formation of micelles starts.

Crafft temp: The temp. above which micelle formation begins.

Micelle — A spherical molecule of soap containing non-polar part towards the centre and polar part outside is called micelle.
Bredie's Arc method.
A method which involves disintegration and condensation by an electric current to form an alloy is called Bredie's Arc method.
All the metallic rods are prepared by this method.

Peptization: A process by which a freshly prepared precipitate is changed into a colloidal sol by shaking it with an appropriate dispersion medium in the presence of a suitable electrolyte is called peptization.

Dialysis: It is the process of removing a dissolved substance from a colloidal solution by means of diffusion through semipermeable membranes.